MSE Minimization and Fault-Tolerant Data Fusion for Multi-Sensor Systems

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Abstract— **Multi-sensor data fusion is an efficient method to provide both accurate and fault-tolerant sensor readouts. Furthermore, detection of faults in a reasonably short amount of time is crucial for applications dealing with high risks. In order to deliver high accuracies for the sensor measurements, it is required to perform a calibration for each sensor. This paper focuses on designing a fault-tolerant calibrated multi-sensor system. First, the least squares method is applied to calibrate each sensor using a linear curve fitting function. Next, an analytical technique is proposed to carry out a fault-tolerant multi-sensor data fusion, while minimizing the Mean-Square-Error (MSE) for the final sensor readout. While our data fusion approach is applicable to different multi-sensor systems, the experimental results are shown for 16 temperature sensors, where an environmental thermal chamber was used as the reference model to calibrate the sensors and perform the measurements.**

I. INTRODUCTION

The last decade has witnessed vast applications for sensing and monitoring devices. The current sensing system specifications require high accuracies, as well as tolerance to hardware, external noise, and potential faults [1]. Hence, numerous researches are dedicated to improving such parameters [2][3][4].

The error occurring at a single sensor's readout can be distinguished as a systematic offset and gain as well as a random noise [5]. The systematic offset and gain are deterministic values. However, the random noise, which is often caused by environmental conditions and hardware noise, is time-variant, and is mostly assumed to have a Gaussian distribution [2]. Calibration, which is defined as the process of mapping raw sensor readings into corrected values [6], can be used to compensate the systematic offset and gain. Note that when the gain and offset are both constant values and independent from the sensor measurements, then the calibration is translated into a linear curve-fitting function.

Multi-sensor data fusion is another efficient approach, which combines data from multiple sensors to achieve more accurate readouts compared to the case where a single sensor is used alone [7]-[10]. A straightforward approach to increase the accuracy of sensor readouts in terms of the error measure Mean-Square-Error (MSE) is to perform an average computation over the results [11]. This technique can improve the MSE by the factor of n, where n is the total number of sensors.

Data fusion methods can also be used to detect faulty sensors [3] and deliver fault-tolerant measurements. This is crucial, since the technology trends indicate that sensing has by far the highest fault rates [12]. Furthermore, fault tolerant sensor readout in a reasonably short amount of time is a critical task for applications dealing with high risks. There exist in the literature approaches that aim to detect the faults in sensor readouts based on the typical fault models in [13]-[15]. Such approaches are based on capturing the sensor readouts for a reasonably long amount of time, and then comparing the results with the given statistical characteristics of the sensors in their normal operation mode. Next, if the results of a sensor deviate from the expected characteristics by a particular threshold, then the sensor is assumed to be faulty [13]. The approach in [13] also proposes a technique to find the optimal threshold value for such a purpose. Albeit useful, these techniques are not applicable to the applications, where it is required to detect the faulty sensors in a short amount of time, *e.g.*, in health systems [3]. In fact, sensor failures may occur within relatively short intervals [3]. Hence, it is a must to provide data fusion techniques to detect faults periodically within a relatively short amount of time. The approach in [3] is useful for such purposes.

In this paper we focus on designing a fault-tolerant calibrated multi-sensor system. Since industrial applications have high demands for the accuracy of temperature measurement and control [16], we particularly, target a practical multi-sensor system consisting of STTS751 temperature sensors. First, the least squares method is applied to calibrate each sensor using a linear curve fitting function with respect to the Temptronic TP4500 environmental thermal chamber as the reference model. Statistical characteristics of the sensor errors are then derived from experimental measurements. Next, an analytical technique is proposed to perform a fault-tolerant multi-sensor data fusion, while minimizing the Mean-Square-Error (MSE) for the final sensor readout. We make use of the fault model in [4]. Namely, we assume a uniform stuck-at fault plus a Gaussian noise, which is derived by the experimental

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measurements from STTS751 temperature sensors. The proposed data fusion technique is based on an average computation over the sensor readouts, where a certain sensor's readout is ignored, when not only it is furthest from the average of other sensor readouts, but also the deviation must be higher than a threshold. We aim to find the optimal threshold, such that the MSE at the final output (average of readouts) is minimized over a wide range of probabilities for a sensor to completely fail. Note that the optimal threshold is a function of the statistical characteristics of the sensors, which are derived from experimental measurements in this paper. Our data fusion approach combines the fault-tolerant and quick fault-detection advantages of the method in [3], and the efficient MSE (high accuracy) of the normal average computation [11], when no sensor is faulty. It is also notable that although we only focus on temperature sensors in this paper; the proposed fault-tolerant data fusion technique is generic and applicable to other multi-sensor systems as well, as long as the statistical characteristics of the sensors are available.

The rest of the paper is organized as follows. Section II briefly discusses the related work. Section III presents the particular system hardware configuration that we use in this paper, as well as the procedure of its calibration. Section IV addresses the proposed analytical framework to perform fault-tolerant multi-sensor data fusion, while minimizing MSE. Experimental results are presented and discussed in Section V. Finally Section VI concludes the paper.

II. RELATED WORK

In this section we address the related work on accuracy analysis, calibration, and fault-tolerant data fusion techniques in multi-sensor systems.

Calibration as a means to increase the accuracy of sensor readouts has been extensively studied in the literature. In [17], Bychkovskiy et al. suggest a methodology for localized calibration of the light sensors. It first considers the physically close sensors and then tries to obtain the most consistent way to provide all pair-wise relationships. J. Feng et.al [2], have focused on a time-variant actuator-based method in order to calibrate the sensors. The results are shown on a set of light intensity measurements recorded by deployed sensors. There are two scenarios in this study, one where only two neighboring sensors have to communicate to achieve the calibrated readouts and the other where a provably minimum number of communications is achieved. In [10], authors consider four types of temperature sensors. They present an analysis on the biases and errors of the specified temperature sensors, signal conditioning circuitry, and data achievement system [10].

The approach described in [16] introduces a system of temperature sensors, which adopts ARM processor LM3S1138 as its controlling core. The Least Squares Method is applied to fit the *sampling value - temperature* [16] curve in connection with the readouts from the temperature sensors. In this method the maximum and minimum readouts of ten measured data at each sampling point are extracted, and then the average of the remaining readouts of the eight sensors is used as the reference model for the purpose of calibration.

A straightforward data fusion approach to increase the accuracy of sensor readouts in a multi-sensor system is to perform a simple average computation over the results [11]. This technique improves the Mean-Square-Error (MSE) by the factor of n, where n is the number of sensors measuring the same data. However, the normal average computation is not robust against faults.

The approaches that deal with the detection of faulty sensors are often based on capturing the sensor readouts for a reasonably long amount of time, and then comparing the results with the given statistical characteristics of the sensors in their normal operation mode. Next, if the results of a sensor deviate from the expected characteristics by a particular threshold, then the sensor is assumed to be faulty [13]. Such techniques cannot detect the potential faulty sensors within a short period of time. In [3] authors present a fault-tolerant technique for glucose sensor readouts based on an average computation over multiple sensor readouts. The basic idea is to throw away the sensor readout, which is furthest from average. That way, faults caused by the complete failure of a sensor, body rejection, etc, can be detected relatively fast to increase the robustness of the average computation. The major inefficiency of this approach is that it does not provide a high accuracy (low MSE) for the final sensor readout, when none of the sensors is faulty.

The proposed data fusion technique in this paper, which is evaluated on the STTS751 temperature sensors, not only aims to minimize MSE at the terminal output (optimizing the accuracy of the whole system), but also to detect the potentially faulty sensors very quickly (within each multiple-sensor readout).

III. SYSTEM CONFIGURATION AND CALIBRATION

In this section we discuss the hardware configuration of the multi-sensor system used in this paper. Furthermore, the calibration technique that is carried out for the individual temperature sensors is explained.

A. System hardware configuration

The STTS751 is a 6-pin digital temperature sensor that supports different slave addresses. The block diagram of STTS751 is shown in Fig. 1 [18]. The STTS751 communicates over a 2-wire serial interface compatible with the SMBus 2.0 standard. Temperature data, alarm limits and configuration information are communicated over the bus. The STTS751 is available in two versions. Each version has 4 slave addresses determined by the pull-up resistor value connected to the Addr/Therm pin. In our experiments, the configurable temperature reading precision is set to 12 bits, or 0.0625 ℃ per LSB.

The data of eight temperature sensors is collected by an STM32F407 microcontroller, which uses an ARM Cortex-M4 32-bit [16] core, as the I^2C master. To accommodate 8 sensors with only four distinct addresses, a second I^2C bus is used. The schematic of the proposed design is shown in Fig. 2. In order to have more data for calibration and experiments, two systems with the mentioned design characteristics are considered.

B. Calibration Algorithm

The least squares method is applied to calibrate each sensor using a linear curve fitting function with respect to the Temptronic TP4500 environmental thermal chamber as the reference model. The Temptronic TP4500 temperature environmental thermal chamber (shown in Fig. 3) is ideal for lab testing and failure analysis of micro-systems due to its fast temperature transitions and high airflow over its wide operating range between -45 °C and +225 °C. It is also capable of traversing the full range within 12 seconds [19]. The TP4500 works by placing a thermal shroud around a device under test, and directing a flow of air over the sample at a controlled temperature. After collecting raw data from 16 sensors, we have to find a function that provides the mapping from the raw sensor readouts to the reference temperatures.

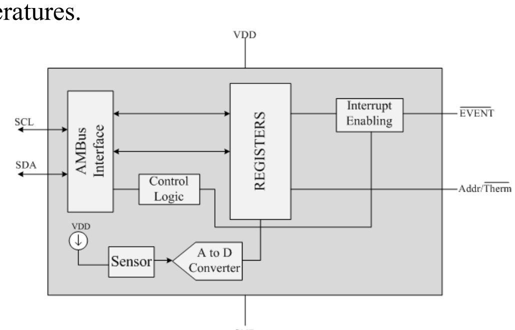

Figure 1. The block diagram of STTS751.

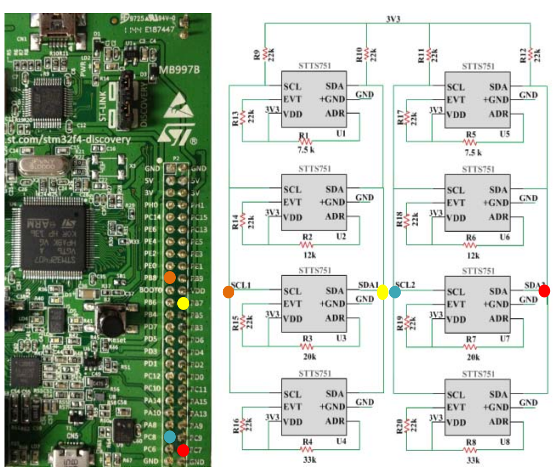

Figure 2. Schematic of the proposed multi-sensor system.

Curve fitting is often used to process data in order to let sensor input-output relationship accord with our requirements [20]. This fitting method makes measured target position closer to its actual position due to the use of Temptronic TP4500 as the reference model.

Linear least-square curve fitting is the most commonly used curve fitting approach, which approximates the output y as a polynomial with respect to the input x, *i.e.*,

$$y = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n. \quad (1)$$

Note that the output y is linear with respect to the coefficients a_0, …, a_n. Regarding sensor measurements, where y is the reference data, and x is the raw sensor readout, a first-degree polynomial, *i.e.*, $n = 1$ in Eq. 1, is mostly sufficient to approximate the output-input characteristics. This is due to the fact that the raw sensor readout mostly involves a constant offset and gain, which can be compensated using a first-degree polynomial. Hence, we make use of the linear least-square curve fitting of the first degree for the calibration of sensors as follows:

$$y = a + bx,$$

where y is the reference data, and x is the raw sensor readouts, while a and b represent the offset and gain, respectively.

The coefficients a and b must be set to minimize the Sum of Square Error (SSE) function, which is defined as follows:

$$SSE = \sum_{i=1}^{N} w_i \left(y_i^{ref} - y_i^{calib}\right)^2,$$

where y_i^{ref} and $y_i^{calib} = a + bx_i$, are the reference and calibrated temperatures at the i^{th} measurement, respectively. The value of x_i is the i^{th} raw sensor readout, and w_i is the weight of each measurement, which accounts for the fact that the standard deviation of the reference temperatures may vary along x. It is calculated as follows:

$$w_i = 1/\sigma_i^2,$$

where σ_i^2 is the variance of x_i.

Figure 3. The Temptronic TP4500 with STTS751 temperature sensors under test.

C. Calibration Results

In our experiment 16 sensors are placed on two $5.2 \times 4.7\ cm$ boards. We assume that after some periods of time the sensor readings become relatively stable in order to conduct the calibration. Twelve minutes for each set point of the Temptronic is considered in order to obtain stable temperature values.

Figure 4 shows our experiment on one of the boards, where the raw temperature readouts and calibrated data are depicted. In this experiment by generating a dual-slope temperature ramp with Temptronic TP4500, the temperatures are changed in 4^o steps between 10^oC and 30^oC with start points 10^oC and 12^oC. It is worth noting that after calibration most of the data converges to the reference temperature as shown in Fig. 5. The results of independently calibrating sixteen STTS751 temperature sensors within the temperature range from 10°C to 30°C are addressed in Table I. Note that the calibration can be performed similarly for other temperature ranges as well. The maximum and minimum absolute values of error before and after calibration are also shown in Table II.

We have also evaluated the confidence level of the calibrated results with respect to different confidence intervals as addressed in Table III. As can be seen the confidence interval of $[-0.3^oC, 0.3^oC]$ covers most of the measurements, *i.e.*, 97.58% of the data.

To provide an acceptable fault-mode for the purpose of fault-tolerant data fusion (which is discussed in Section IV), we require the statistical characteristics of noise. The results shown in Fig. 4 indicate that we can distinguish between the systematic bias/gain error and the random noise component of the error. It is shown in Fig. 4 that within the temperature range of 10°C to 30°C, there exists a small difference between the reference temperature and the calibrated data, which corresponds to the random noise.

We expect the random noise to have a normal distribution. Hence, we have evaluated the difference between reference and calibrated temperatures for as many measurements as we can to get the distribution of this error. Namely, more than 100,000 reference temperature samples have been evaluated for all the 16 temperature sensors.

The results have shown that such an error can be fitted into a normal distribution as depicted in Fig. 6. In this figure the fitted normal distribution corresponds to the log-likelihood value of 150.067, which indicates a good fit. Furthermore, the error measure R-Square, which indicates the goodness of the fit in terms of the variation of data, is also obtained as 98.72% for the normal distribution fit in Fig. 6.

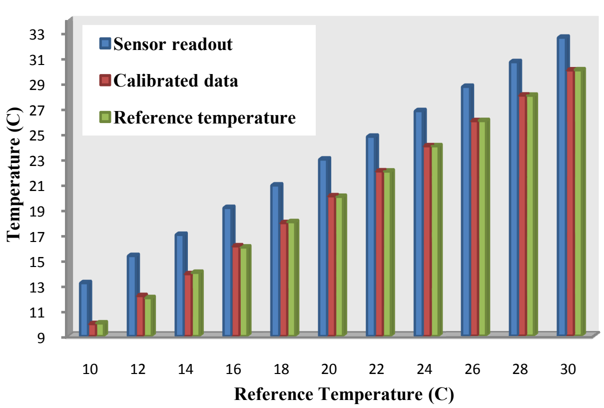

Figure 4. Systematic bias/gain and random noise of the temperature sensor measurements.

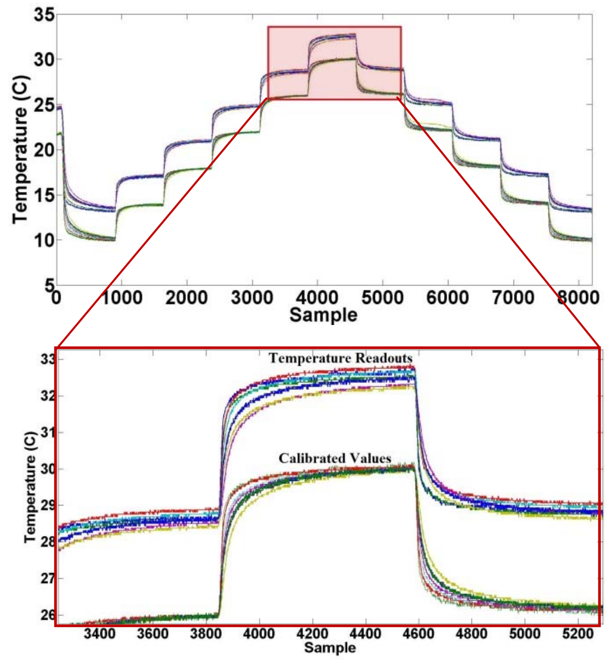

Figure 5. Temperature readouts and calibrated values for eight sensors, when using a dual-slope ramp for the reference temperatures and start point 10^oC.

TABLE I. OPTIMAL COEFFICIENTS TO CALIBRATE OUR SENSORS.

Sensor #	Gain (b)	Offset(a)
1	1.034	-3.684
2	1.042	-3.829
3	1.034	-3.906
4	1.038	-3.901
5	1.053	-4.068
6	1.055	-4.029
7	1.037	-3.701
8	1.044	-3.901
9	1.074	-4.439
10	1.076	-4.61
11	1.063	-4.4
12	1.068	-4.729
13	1.07	-4.691
14	1.074	-4.672
15	1.07	-4.45
16	1.076	-4.685

TABLE II. MAXIMUM AND MINIMUM ABSOLUTE VALUES OF ERROR BEFORE AND AFTER CALIBRATION.

Parameter	Before Calibration	After Calibration
Maximum Absolute Error (^oC)	3.6	0.6535
Minimum Absolute Error (^oC)	2.2	0.0015

TABLE III. CONFIDENCE LEVEL OF THE ERROR OF 16 CALIBRATED SENSORS W.R.T. THE REFERENCE CHAMBER OVER DIFFERENT INTERVALS

Confidence Interval (ºC)	Confidence Level (%)
[-0.1,0.1]	45.32
[-0.2,0.2]	79.15
[-0.3,0.3]	97.58
[-0.4,0.4]	99.4
[-0.5,0.5]	99.4
[-0.6,0.6]	99.7
[-0.6535,0.6535]	100

We apply this normal distribution to represent noise in the fault-model of the proposed fault-tolerant data fusion approach, which is discussed in Section IV.

IV. PROPOSED FAULT-TOLERANT DATA FUSION APPROACH

In this section we present a fault-tolerant data fusion technique for the multi-sensor system described in Section III. The discussion, however, is generic and applicable to other multi-sensor systems as well. The proposed data fusion method is based on an analytical model for the MSE minimization (optimizing the accuracy of the sensor readouts).

A. Statistical Distributions and Assumptions

For the purpose of this analysis, we make use of a number of assumptions:

- *Sensor Normal Operation Mode:* The proposed data fusion approach is based on average computations as well as computing the difference between sensor readouts and the average value. Hence, we have derived the statistical characteristics of the error between the calibrated sensor readout and the average of all readouts from the 16 temperature sensors when they are operating in their normal operation mode, *i.e.*, the sensor is not faulty. The experimental measurements have shown that such an error fits the student's t-distribution (See Fig. 7). The t-distribution is useful when estimating the mean of a normally distributed population, where the sample size is small and population standard deviation is unknown. It is symmetric and bell-shaped, like the normal distribution. However, it has heavier tails, which makes it more inclined to producing values that fall far from its mean. As depicted in Fig. 7, the t-distribution is a better fit to our measurements compared to the normal distribution.
- *Sensor Failure Probability:* Each sensor has a probability of complete failure, *e.g.*, p, where p is a very small value, *e.g.*, $p < 0.01$. Note that since p is a small value, we assume in this paper that only one sensor is probable to completely fail at a time (no multiple faults).
- *Sensor Failure Readout Distribution:* When a sensor completely fails, the sensor readout could be anything. We make use of the fault model in [4] to assume that the sensor readout has a uniform distribution among the effective range of the sensor readouts, *i.e.*, $[10^o, 30^o]$ for the temperature sensors in this paper. In fact, it is assumed that the temperature readout is stuck at a particular temperature. Furthermore, the fault model in [4] requires adding a Gaussian noise to the sensor readout. Therefore, we make use of the Gaussian noise model obtained by our experimental measurements (See Fig. 6) to add to the fault model.
- *Sensor Failure Original Data Distribution:* The original data that the failed temperature sensor is supposed to read could be anything in the range of $[10^o, 30^o]$ as well. In this paper we assume a uniform distribution for the original data among the full-scale. However, other distributions can be chosen for the original data as well. In fact it might be possible that in some particular cases, specific original values are more probable to be read by the sensor.
- *Sensor Failure Error Distribution:* Based on the uniform distributions of the original data and the sensor readout in the failure mode, the sensor error has a triangular distribution in its failure mode within the range of $[20^o, 40^o]$, added by a Gaussian noise.

It is notable that any other statistical distributions considered for the above variables, can also be handled by the proposed analytical framework and MSE optimization in this section.

B. Proposed Data fusion

The proposed fault-tolerant data fusion approach, which is based on an average computation, is presented in this subsection. We assume that n multiple-sensor readouts, *i.e.*, $x_1,\ldots,x_n$, measuring the same thing, are available at a time, and the goal is to return a single readout, $\hat{x}$, such that MSE is minimized and sensor failures are also handled.

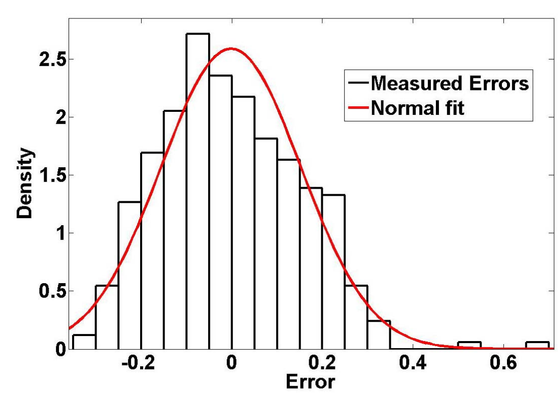

Figure 6. Fitting the error of 16 calibrated sensors to a normal distribution.

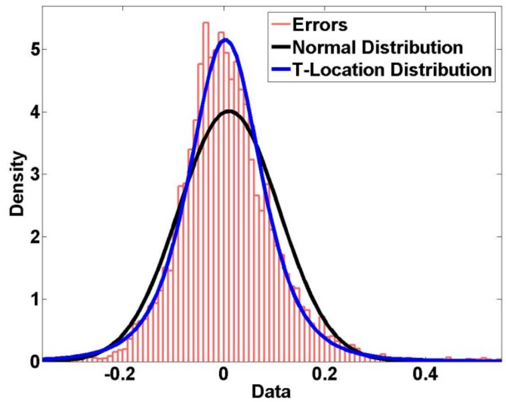

Figure 7. T-distribution and normal fitting of the error between each calibrated sensor and the average of all measurements.

The proposed data fusion algorithm is outlined in Fig. 8. First, corresponding to each sensor readout x_i, an average computation excluding the variable x_i is performed in Steps 1 and 2. Furthermore, the variable d_i computed at Step 3 shows the absolute difference (deviation) of x_i from the average of others. Next, we find the sensor readout resulting in the maximum deviation, *i.e.*, d_i, in Steps 4 and 5. Finally, if the deviation is higher than a given threshold *T*, then we exclude x_i within the final average computation in Step 8, since it is most likely that the sensor is faulty. Otherwise, a normal average computation is performed (when $u_i = 1$). Note that through the rest of this section we aim to find the most suitable value of *T* to minimize the MSE at the terminal output (single measurement $\hat{x}$).

C. MSE Minimization

The error occurring at the final measurement $\hat{x}$ in Fig. 8, as well as MSE can be computed as follows:

$$e_{\hat{x}} = \frac{e_i u_i + \sum_{j=1,j\neq i}^{n} e_j}{n-1+u_i},$$

where e_j $(j = 1, \dots, n)$ is the error of the calibrated j^{th} sensor readout, e_i is the error of the sensor readout, which is furthest from the average of the rest of the sensor readouts (See Step 5 in Fig. 8). Hence, MSE can be obtained as follows:

$$\Rightarrow MSE = E(e_{\hat{x}}^2) = E(\left(\frac{e_i u_i + \sum_{j=1,j\neq i}^{n} e_j}{n-1+u_i}\right)^2)$$

$$= E\left(\frac{e_i^2 u_i^2 + \left(\sum_{j=1,j\neq i}^{n} e_j\right)^2 + 2e_i u_i (\sum_{j=1,j\neq i}^{n} e_j)}{(n-1+u_i)^2}\right)$$

$$\xrightarrow{u_i=0,1} = E\left(\frac{e_i^2 u_i}{(n-1)^2 + u_i(2n-1)}\right) + E\left(\frac{\left(\sum_{j=1,j\neq i}^{n} e_j\right)^2}{(n-1)^2 + u_i(2n-1)}\right)$$

$$+2E\left(\frac{e_i u_i (\sum_{j=1,j\neq i}^{n} e_j)}{(n-1)^2 + u_i(2n-1)}\right), \quad (2)$$

where $E(.)$ is the expectation function. Furthermore, based on the code in Fig. 8 we have:

$$u_i = \begin{cases} 1 & d_i \leq T \\ 0 & otherwise \end{cases}, \quad (3)$$

where

$$d_i = \left| x_i - \frac{\sum_{j=1,j\neq i}^{n} x_j}{n-1} \right|. \quad (4)$$

The statistical characteristics of d_i depends on whether the i^{th} sensor is faulty or not. Hence, we re-write d_i as

$$d_i = p_i d_{failure\_i} + (1 - p_i) d_{normal\_i}, \quad (5)$$

where p_i is the probability of Sensor#i to completely fail, while d_{normal_i} and $d_{failure\_i}$ are the values of errors d_i in the normal and complete failure modes, respectively. According to the discussion in Subsection IV-A, d_{normal_i} has a student's t-distribution (See Fig. 7), while $d_{failure\_i}$ has a triangle-like distribution over the range of $[20^o, 40^o]$, which is also added by the Gaussian noise in Fig. 6.

Eqs. (2) to (5) indicate that there is a complex correlation between u_i, d_i, and e_j $(j = 1, \dots, n)$. Hence, computing the MSE in Eq. 2 analytically requires knowing the value of p_j in Eq. 5 first and then solving a complicated n-dimensional joint probability distribution integral corresponding to d_j $(j = 1, \dots, n)$, which becomes hard as n increases. However, as typically the value of n (number of multiple sensor readouts) is small, we aim to compute an almost accurate value of MSE in Eq. 2 by making use a numerical method. Particularly, we divide the input intervals into a number of smaller ones and then compute the integral by doing the summation instead. We then gradually increase the number of intervals, *e.g.*, by multiplying it by two after each iteration, until the computed value of MSE converges. Please note that the run-time is not an issue, since these computations unlike the data fusion algorithm in Fig. 8 are performed offline and only once to compute MSE. Next, on the top of the MSE computation using the Newton method, we aim to find the optimal threshold *T* to minimize the MSE in Eq. 2. The proposed algorithm to set the threshold value *T* is shown in Fig. 9. The inputs to the algorithm are the statistical characteristics of $d_{failure\_1}$, …, $d_{failure\_n}$, d_{normal_1}, …, d_{normal_n}, while the outputs are the optimal threshold *T*, *MSE_no_fault*, which is the MSE in Eq. 2 when no fault occurs, *i.e.*, when $p_j = 0$. In Eq. 5 $(j = 1, \dots, n)$, as well as *MSE_fault*, which is the MSE in Eq. 2 when only one sensor, *e.g.*, Sensor#k $(k \in \{1, \dots, n\})$, fails $(p_k = 1, p_{j=1,\dots,n,j\neq k} = 0)$. Note that other heuristics could be used instead of the Newton method to find the optimal threshold *T* as well.

V. Results

In this section we present the results on the MSE minimization using the algorithms in Fig. 8 and Fig. 9 compared to the normal average computation [11], Average, and the fault-tolerant data fusion technique in [3]. The algorithms have been implemented in MATLAB and executed on an Intel 2.1 GHz Core 2 and 2 GByte running under Windows XP.

```
Single_Measurement (x_{1:n}, x̂, T)
//Inputs: sensor readouts x_{1:n}, the Threshold T, Output: x̂
1- for (i = 1; i ≤ n; i++)
2- { a_i = (Σ_{j=1,j≠i}^{n} x_j)/(n−1); //Average computation excluding x_i
3-   d_i = |x_i − a_i|; } //Deviation from the average of others
//Find the furthest from average of others
4- for (i = 1; i ≤ n; i++)
5- { if d_i = max(d_{1:n}) break; }
6- if d_i > T  u_i = 0; //Throw away x_i
7- else u_i = 1; //Do not throw away x_i
8- return x̂ = (x_i u_i + Σ_{j=1,j≠i}^{n} x_j)/(n−1+u_i); //Final average computation
```

Figure 8. Proposed data fusion algorithm.

```
Threshold_Opt d_{failure_1},…,d_{failure_n}, d_{normal_1}, …,
  d_{normal_n}, MSE_no_fault, MSE_fault, T)
//Inputs: e_{failure_1},…,e_{failure_n}, e_{normal_1}, …,e_{normal_n}
//Outputs: MSE_no_fault, MSE_fault, T
// MSE_no_fault is the value of MSE when: p_{1:n} = 0
// MSE_fault is MSE when: p_{j=1,…,n,j≠k} = 0, p_k = 1
1- Set an initial T;
2- Find the optimal T, such that:
        3- MSE_no_fault ≈ MSE_avg,
        4- MSE_fault = minimized,
/*MSE_avg is the MSE of normal average computation when
  p_{i∈{1,…,n}} = 0 */
```

Figure 9. Finding the optimal threshold for the data fusion algorithm.

In the first experiment we look into the MSE of the proposed data fusion compared to the normal average computation and the approach in [3] over different probabilities for a sensor to completely fail, *i.e.*, the value of p_j in Eq. 5. We have swept the value of p_j from 0 to 0.01, and the results are shown in Fig. 10, where the number of sensors is set to 3. It is also notable that while the algorithm in Fig. 9 is performed only once and offline to find the MSE and the optimal threshold T, the data fusion technique in Fig. 8 has to be performed online and periodically. As shown in Fig. 10 the proposed fault-tolerant data fusion technique improves the MSE obtained by the approach in [3] by 34% in average. Furthermore, the optimal threshold in Fig. 9 is obtained as $T = 0.235^oC$. The aforementioned experiment resulted in the following interesting observation:

Observation: *If the sensors are identical in terms of probability distributions and statistical characteristics, the optimal threshold obtained by the algorithm in Fig. 9 is independent from the number of sensors.*

Hence, if the statistical characteristics of the sensors are identical to each other, which is mostly the case, *e.g.*, in our system configuration, the optimal threshold obtained for the case with $n = 3$ (which can be obtained relatively fast according to the reduced dimensions for the computation of the integrals and MSE), can be used as the optimal threshold

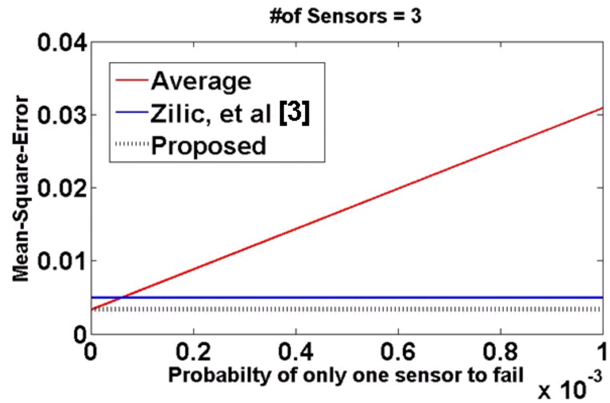

Figure 10. MSE analysis of three data fusion techniques versus different complete failure probabilities.

for the cases with $n > 3$ as well. An example is our multi-sensor system, which involves 16 temperature sensors.

In the second experiment we address the probabilities for a wrong fault detection using the proposed data fusion. Note that regarding the normal average computation, since faults are not being detected, there is no wrong detection. Furthermore, the approach in [3] is based on throwing away the sensor readout that is furthest from the average of others. Hence, the probability of wrong detections is equal to 1, when all the sensors are working fine. Even when the probability of a sensor to completely fail is relatively high, *e.g.*, $p_j = 0.1$, the probability of a wrong fault detection for the approach in [3] is still high (more than 80% when $p_j = 0.1$). We evaluate the probability of a wrong fault-detection for the proposed method, when the number of sensors n is set to 16, which corresponds to our system of 16 temperature sensors. The t-distribution derived from the experimental measurements, which is shown in Fig. 7, is used for this purpose. Figure 11 highlights the wrong decisions when the sensors are in their normal operation mode (no faults) with $T = 0.235^oC$. Table IV presents the final results including the probability of a wrong fault-detection with respect to the probability of a single sensor to fail, *i.e.*, p_j. Note that since the proposed data fusion approach is performed periodically, it can resolve its previous wrong decisions within the next measurements.

VI. Conclusion and Future Work

In this paper we presented a fault-tolerant data fusion technique aiming to minimize the MSE of sensor measurements in a calibrated multi-sensor system. The approach makes it possible to not only detect the potentially faulty sensors in a short amount of time, but also to improve the accuracy of the final measurement when all the sensors are in their normal operation mode.The proposed generic data fusion approach is particularly evaluated on a system of 16 temperature sensors. First, the least squares method is applied to calibrate each sensor using a linear curve fitting function with respect to an environmental thermal chamber

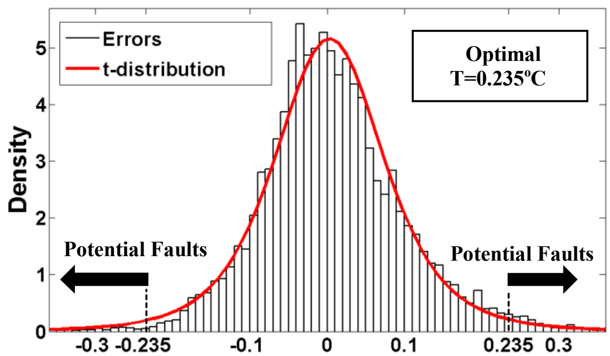

Figure 11. Finding the potential faulty sensors using the t-distribution derived from experimental measurements as well as the the optimal threshold of $T = 0.235^oC$.

TABLE IV. WRONG FAULT DETECTION PROBABILITY W.R.T. p_j FOR THE SYSTEM OF 16 TEMPERATURE SENSORS

p_j	Probability of wrong fault detections
0	0.1038×10^{-5}
0.001	0.1037×10^{-5}
0.01	0.1027×10^{-5}
0.1	0.0934×10^{-5}

as the reference model. Next, the statistical characteristics of the calibrated sensors are computed using thousands of measurements. Finally, the proposed data fusion algorithm is implemented for the system to deliver both low MSE (high accuracy) as well as high fault-tolerance. Experiments compare the efficiency of the proposed data fusion technique in terms of MSE with the normal average computation [11] and the data fusion technique presented in [3].

The proposed data fusion approach is not only suitable to detect the faulty sensors relatively fast, but also to provide high accuracies. Hence, it would be a promising future work avenue to investigate the proposed data fusion algorithm for applications that deal with high risks such as patients' health. For instance, regarding closed-loop insulin control systems for managing glucose levels, sensor readouts should not only have a high accuracy, but also must be robust enough to detect the faulty sensors in a reasonably short amount of time, such that the continuously injected insulin does not bring the patient into the possibly dangerous state of hypoglycemia. Handling multiple-faults and dealing with sensor unknowns as don't cares including using algebraic methods for optimization [21] could be done as the next step.

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